Simplified Restrainer Design Procedure for Multiple-Frame Bridges

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The collapse of existing bridges due to unseating at supports and intermediate (in-span) hinges with inadequate seat-width can be prevented by the use of restrainers to limit the relative hinge opening. A new simplified procedure for the design of restrainers in bridges accounts for the dynamic characteristics and out-of-phase motion of adjacent frames as well as the inelastic behavior of the bridge. The simplified procedure is developed from an empirical relationship for the restrainer stiffness as a function of the frame stiffnesses, initial hinge displacement, target displacement, and target ductility of the structure. Parameter studies and case studies show that the simplified procedure limits the relative hinge displacement to a designer-specified value. [DOI: 10.1193/1.1423652]

INTRODUCTION

Recent earthquakes have demonstrated the vulnerability of bridges to major damage during strong ground shaking (Brunsdon et al. 2000, Kawashima et al. 1996). An undesirable cause of collapse is unseating of bridge decks at the intermediate hinges or at end supports with inadequate seat width. When the relative displacement at the deck exceeds the support at the intermediate hinge or abutment, the deck can unseat and collapse. New bridges are designed with large enough seat width to avoid unseating. However, existing bridges typically do not have adequate seat widths. To reduce the risk of collapse due to unseating, existing bridges are typically retrofitted with restrainers at the intermediate hinges and/or abutments, as shown in Figure 1. Because of their low cost and ease of installation, restrainers are one of the most commonly used retrofit measures for multiple-frame bridges.

After the collapse of several bridges in the 1971 San Fernando earthquake due to unseating, the California Department of Transportation (Caltrans) initiated a retrofit program to install cable restrainers at intermediate hinges in bridges with short seat widths. Although California has by far led the nation in retrofits using cable restrainers, many other states have begun bridge retrofit initiatives, which include providing cable restrainers for vulnerable bridges. The departments of transportation in Washington, Oregon, Utah, Missouri, Tennessee, South Carolina, and Illinois have numerous bridges scheduled for retrofit with cable restrainers (Lam 2000).

The 1989 Loma Prieta earthquake caused numerous cases of restrainer damage and two cases of restrainer failure (Saiidi et al. 1993). During the 1994 Northridge earthquake several bridges that had been retrofitted with cable restrainers collapsed due to unseating at the hinges (Moehle 1995), notably the Gavin Canyon bridge on Interstate 5.

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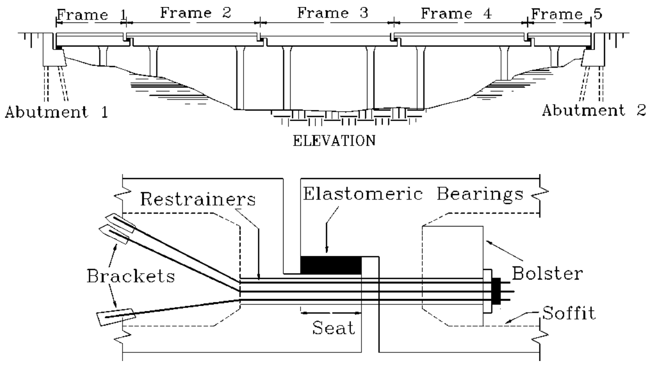

Figure 1. Typical multiple-frame bridge with intermediate hinge restrainers.

During the 1995 Kobe earthquake, many bridge structures in Kobe were damaged. A major problem was excessive movement at the hinges due to bearing and restrainer failure (Comartin et al. 1995).

Although hinge restrainers appear to have been effective in providing protection for multiple-frame bridges, recent earthquakes illustrate that more research is required to develop reliable restrainer design procedures. Previous studies have shown that different design methods produce considerable differences in the number of restrainers selected for a particular bridge (Trochalakis et al. 1996, DesRoches and Fenves 2000). Current restrainer design procedures either do not adequately represent the dynamic interactions of bridge frames or require iteration.

The objective of this paper is to present a design procedure for multiple-frame bridges that is simple and yet represents the important factors affecting the earthquake response of bridge frames connected with hinge restrainers. The effect of skew, curved bridges, and multi-support excitation are not considered in this paper.

CURRENT DESIGN METHODS FOR HINGE RESTRAINERS

Two procedures are used to design restrainers in the United States: the Caltrans (1990) Equivalent Static Procedure, and the AASHTO (1992) procedure. Other procedures have been proposed, including Trochalakis et al. (1997), DesRoches and Fenves (2000), and Priestley, Seible, and Calvi (1996). A comparison of these procedures along with the Japanese restrainer design procedure is presented in this section.

EQUIVALENT STATIC METHOD

The California Department of Transportation (Caltrans) uses an equivalent static procedure for designing hinge restrainers (Caltrans 1990). The procedure considers the frame on the side of the hinge that has the smallest displacement under a lateral earthquake load, evaluated from the earthquake response spectrum. Cable restrainers are provided until the frame displacement is less than the available seat width. The procedure uses the smallest displacement of the two frames, which can lead to an unconservative estimate of the relative hinge displacement. For the case of a very stiff frame adjacent to a very flexible frame, the relative hinge displacement predicted by the Caltrans procedure would be very small. The more flexible frame, however, will likely control the relative hinge displacement. In general, the Caltrans procedure is unconservative for out-of-phase frames (frames with large differences in vibration period), and conservative for in-phase frames (frames with period ratios approaching unity) (Yang et al. 1994, DesRoches and Fenves 2000).

AASHTO

The American Association of State, Highway, and Transportation Officials specification requires a positive horizontal linkage between adjacent frames of the superstructure (AASHTO 1992). The linkage force is equal to the design acceleration coefficient multiplied by the weight of the lighter of the two adjoining frames. The AASHTO procedure has similar shortcomings to the Caltrans procedure in that the linkage force required is based only on the response of one frame. There is no consideration of the relative hinge displacement, which has been shown to be an important parameter (DesRoches and Fenves 2000, Trochalakis et al. 1997).

JAPAN

The Japanese bridge design specifications for restrainers are similar to AASHTO. They require a horizontal restrainer force between frames equal to twice the vertical reaction at the hinge multiplied by the design acceleration coefficient (Takahashi 1990). Similar to the AASHTO procedure, the Japanese specifications do not consider the relative displacement at the hinge.

TROCHALAKIS et al. (1997)

Trochalakis et al. (1997) proposed a restrainer design procedure similar to the Caltrans procedure in that it considers two uncoupled single-degree-of-freedom systems. Using the independent frame displacements, the maximum relative hinge displacement is estimated from a regression of the response data from nonlinear dynamic analyses. The following expression for the relative hinge opening was proposed:

$$D_{eq} = \frac{D_{ave}}{2} \frac{T_L}{T_S} \leqslant 2D_{ave} \tag{1}$$

where D_{ave} is the average independent frame displacement, and T_L and T_S are the longer and shorter periods of the individual bridge frames. The periods are calculated by assuming the restrainers are fixed at one end and attached to the frame at the other end. This procedure is an iterative procedure. In each step the restrainer stiffness determines the periods, T_L and T_S, which in turn determines the average frame displacements (D_{ave}) and the relative hinge displacement (D_{eq}). Results from dynamic analyses show that this procedure provides a better estimate of the required restrainer stiffness to limit hinge displacement than the AASHTO or equivalent static procedures.

DESROCHES AND FENVES (2000)

DesRoches and Fenves (2000) proposed an iterative restrainer design procedure for multiple-frame bridges. This procedure represents the dynamic characteristics and out-of-phase motion of adjacent bridge frames, including the inelastic behavior of frames. Two-degree-of-freedom modal analysis is performed to determine the relative hinge displacement. The hinge displacement, D_{eq}, is estimated by combining the modal response using the complete quadratic combination (CQC) rule (Der Kiureghian 1980):

$$D_{eq}=\sqrt{D_1^2+D_2^2+2\rho_{12}D_1D_2} \tag{2}$$

where D_1 and D_2 are modal hinge displacements and ρ_{12} is the correlation coefficient between the response of the two modes defined as:

$$\rho_{12}=\frac{8\sqrt{\xi_1\xi_2}(\xi_1+\beta\xi_2)\beta^{3/2}}{(1-\beta^2)^2+4\xi_1\xi_2\beta(1+\beta^2)+4(\xi_1^2+\xi_2^2)\beta^2} \tag{3}$$

where β is the ratio of the frame periods, T_2/T_1, and ξ_1 and ξ_2 are the modal damping ratios.

Using D_{eq}, the restrainer stiffness, K_r, needed to limit the hinge displacement to the maximum specified hinge displacement, D_r, is determined from the sensitivity of the hinge displacement to the restrainer stiffness:

$$\frac{\partial D_{eq}}{\partial K_r}=-\frac{1}{K_m+K_r}D_{eq} \tag{4}$$

where $1/K_m=1/K_1+1/K_2$ is the sum of the flexibilities of the two frames. Performing a Taylor series expansion about the current estimate of the hinge displacement, D_{eq_j}, and solving for an improved estimate of the restrainer stiffness at the next step, $K_{r_{j+1}}$, gives

$$K_{r_{j+1}}=K_{r_j}+(K_m+K_{r_j})\frac{(D_{eq_j}-D_r)}{D_{eq_j}} \tag{5}$$

Each iteration of the procedure consists of a 2-DOF modal analysis for D_{eq}, followed by the use of the updated estimate of restrainer stiffness. The yielding behavior of ductile frames is accounted for in the restrainer design procedure by determining an equivalent stiffness and damping ratio based on the maximum displacement of the frames (Gulkan and Sozen 1974). The typical case requires three to five iterations of the procedure to converge to the restrainer design. Parameter studies and case studies show that the procedure limits the relative hinge displacement to a specified value for a wide range of bridges (DesRoches and Fenves 2000).

The Caltrans (1990), Trochalakis et al. (1997), and DesRoches and Fenves (2000) procedures require several iterations to converge to a restrainer design. There is a need for a simplified restrainer design procedure that does not require iterations and yet provides a reliable estimate of the required number of restrainers to limit the hinge displacement to a prescribed value.

SIMPLIFIED DESIGN PROCEDURE

A restrainer design procedure for multi-frame bridges is developed that does not require iteration. The simplified procedure is developed from an empirical relationship for the restrainer stiffness as a function of the frame stiffnesses, initial hinge displacement, target displacement, and target ductility of the frames.

NORMALIZED RESTRAINER STIFFNESS

Results from the multiple-step restrainer design procedure show that the restrainer stiffness to limit the hinge displacement is primarily a function of the composite frame stiffness, K_m, the target ductility of the frames, μ, the unrestrained relative hinge displacement, D_{eq_0}, and the specified hinge displacement, D_r (DesRoches and Fenves 2000). Based on these observations a convenient and physically meaningful normalization for restrainer stiffness is:

$$\underline{K} = \frac{K_r\, D_r}{\dfrac{K_m}{\mu}\left(D_{eq_0} - D_r\right)} \tag{6}$$

The numerator of Equation 6 represents the desired restrainer force required to limit the hinge displacement to the target displacement, D_r. The denominator represents the force required to slowly pull the frames together from an initial opening of D_{eq_0} to a final opening of D_r, acting through an effective stiffness, K_m/μ, of the two frames. Rewriting Equation 6 and solving for K_r the required restrainer stiffness to limit hinge displacement can be expressed in terms of the normalized restrainer stiffness:

$$K_r = \frac{K_m}{\mu}\left(\frac{D_{eq_0}}{D_r} - 1\right)^{*} \underline{K} \tag{7}$$

From Equation 7, it is seen that the restrainer stiffness required to limit hinge displacement is a function of three parameters: K_m/μ, D_r/D_{eq_0}, and $\underline{K}$. DesRoches and Fenves (1997b) have shown that the nondimensional stiffness ratio, $\underline{K}$, is a function of the frame period ratio (T_1/T_2), target ductility (μ), displacement ratio (D_r/D_{eq_0}), and the ground motion period ratio ($\underline{T}_g = T_{2\,\mathrm{eff}}/T_g$), where T_g is the characteristic period of the ground motion, and the effective period of the frame with longer period is $T_{2\,\mathrm{eff}} = T_2\sqrt{\mu}$. The characteristic period of the ground motion, T_g, is based on the peak of the pseudo-velocity response spectrum (Uang and Bertero 1990). Equation 7 accounts for the effects of yielding frames through the effective modified frame stiffness, K_m/μ.

The normalized restrainer stiffness, $\underline{K}$, is evaluated as a function of the controlling factors described in the previous section. The value of the actual restrainer stiffness required to limit hinge displacements is determined from the multiple-step restrainer design procedure (DesRoches and Fenves 2000). A suite of 26 strong earthquake ground motion records shown in Appendix B (Table B1) are used to determine how the normalized stiffness varies over the range of parameters discussed above. The historical records are scaled to a peak ground acceleration of 0.70g and are selected to represent a wide

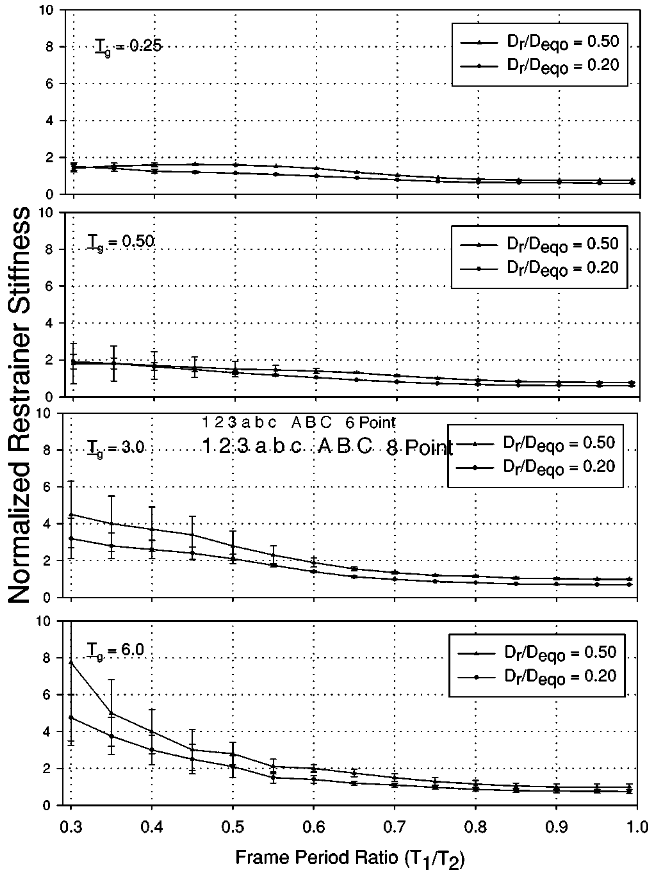

Figure 2. Mean +/− one standard deviation of the normalized restrainer stiffness for 26 ground motion records for D_r/D_{eq_0}=0.20, and 0.50, T_g=0.25, 0.50, 3.0, and 6.0, and μ=1, 4, and 6.

range of characteristic periods, peak ground accelerations, epicentral distance, and duration. Figure 2 shows the value of the normalized restrainer stiffness for T_g values of 0.25, 0.50, 3.0, and 6.0, and μ values of 1, 4, and 6, as a function of the frame period ratio, T_1/T_2. The normalized restrainer stiffness decreases as the frame period ratio increases. As the frame period ratio approaches unity, the participation factor for relative hinge displacement for modes 1 and 2 decreases to zero, which reduces the required restrainer stiffness. As shown in Figure 2, the normalized stiffness increases with increasing T_g. For large T_g, the effective period of the frame is greater than the effective period of the ground motion. As restrainers are added, the vibration period decreases, increasing the seismic forces. Finally, Figure 2 shows that increases in D_r/D_{eq_0} lead to a slight increase in the normalized restrainer stiffness.

The variability in normalized restrainer stiffness, as measured by the standard deviation, increases with decreasing T_1/T_2, increasing T_g, and increasing D_r/D_{eq_0}. The results of analysis from 26 ground motion records show that the expression for the normalized restrainer stiffness is strongly dependent on the ground motion period ratio, frame period ratio, and the target displacement ratio for frame stiffness ratios $\mathrm{T}_1/\mathrm{T}_2<0.60$. In this range, the equation for the normalized stiffness can be written as:

$$\underline{K}=\underline{D}\left[2+0.30\underline{T}_g-(4.9+\underline{T}_g)\left(\frac{T_1}{T_2}-0.30\right)\right] \qquad \left(\frac{T_1}{T_2}<0.60\right) \tag{8}$$

where

$$\underline{D}=\left[1+1.66\left(\frac{D_r}{D_{eq_o}}-0.20\right)\right] \tag{9}$$

For the range of frame stiffness ratios $\mathrm{T}_1/\mathrm{T}_2\geqslant 0.60$, the variability in the normalized restrainer stiffness is small and does not vary much as a function of $\mathrm{T}_1/\mathrm{T}_2$ and $\underline{\mathrm{T}}_g$. In the range $\mathrm{T}_1/\mathrm{T}_2\geqslant 0.60$, $\underline{K}$ is fairly constant, with slight increases for larger D_r/D_{eq_0}. Based on this, the expression for the normalized restrainer stiffness is written as:

$$\underline{K}=\frac{D_r}{D_{eq_o}}+0.50 \qquad \left(0.60\leqslant\frac{T_1}{T_2}<1.00\right) \tag{10}$$

It is observed from Equation 10 that the normalized restrainer stiffness increases as D_r/D_{eq_0} increases. However, this does not lead to increases in the restrainer stiffness. Substituting Equation 10 into Equation 7 produces the following expression for the restrainer stiffness as a function of the D_r/D_{eq_0} ratio:

$$K_r=\frac{K_m}{\mu}\left[0.50+\frac{0.50-\left[\dfrac{D_r}{D_{eq_0}}\right]^2}{\dfrac{D_r}{D_{eq_0}}}\right] \qquad \left(0.60\leqslant\frac{T_1}{T_2}<1.00\right) \tag{11}$$

From Equation 11, as D_r/D_{eq_0} approaches zero, the restrainer stiffness approaches infinity. This represents the case of limiting the hinge displacement to zero. As D_r/D_{eq_0} approaches unity, the restrainer stiffness approaches zero. The latter represents the case where the target displacement is equal to the hinge opening without restrainers.

EVALUATION OF SIMPLIFIED EXPRESSION FOR RESTRAINER STIFFNESS

The effectiveness of the simplified expression for the restrainer stiffness was evaluated for a range of period ratios and design ductilities using the 26 strong earthquake ground motion records discussed above. For each case, a nonlinear time history analysis was performed using the restrainer stiffness determined from the procedure. The model for the nonlinear analysis consisted of two single-DOF systems, each representing the longitudinal response of a bridge frame. The frames were modeled using the Q-hyst stiffness degrading hysteretic relationship (Gulkan and Sozen 1974). The restrainers were modeled as bilinear springs in tension. Impact between frames was modeled using

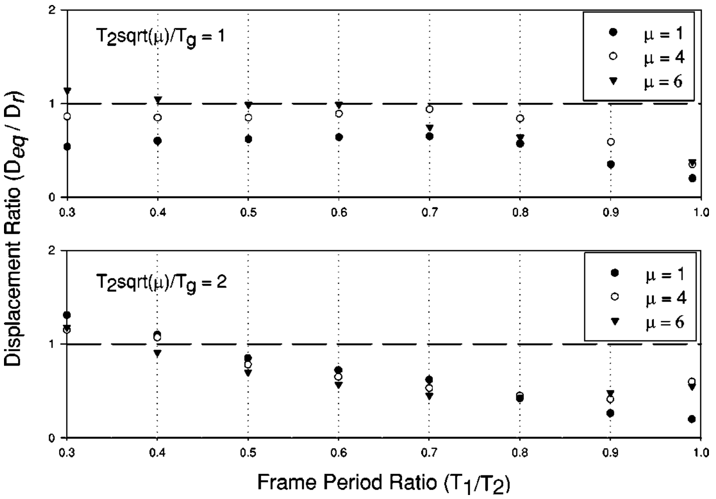

Figure 3. Mean ratio of hinge displacement from nonlinear analysis divided by target displacement for 26 ground motion records, $D_r/D_{eq_0}=0.50$.

the stereomechanical approach (DesRoches and Fenves 1997b). The equations of motion are solved numerically using Newmark's constant acceleration method.

The results of the nonlinear dynamic analyses are presented in terms of the ratio of maximum hinge displacement from nonlinear analysis, D_{eq}, to target displacement, D_r, for the suite of 26 strong ground motion records. Figure 3 shows the response ratio for $D_r/D_{eq_0}=0.50$, and $T_2\sqrt{\mu}/T_g$ values of 1.0 and 2.0 for T_1/T_2 ranging from 0.30 to 1.0. For frame period ratios $T_1/T_2 \geqslant 0.60$, the mean of the ratio for $\mu=1,4$, and 6 is less than unity, indicating that the procedure is conservative in this range. This is expected since the nonlinear analysis includes the effects of pounding between the frames, which typically reduces the hinge displacement for cases where T_1/T_2 approaches unity. In general, there is no difference in the effectiveness of the procedure for the three ductility levels. For $T_1/T_2<0.60$, the displacement ratio is slightly greater than unity, and increases for decreasing values of T_1/T_2 and increasing values of T_g. Pounding is pronounced in out-of-phase frames and tends to increase the response for low period ratios. Because of the variability in normalized stiffness and increasing displacement ratio for $T_1/T_2<0.60$, the proposed simplified design procedure should be limited to frame period ratios greater than 0.60. This allows the direct use of the simplified expression for the restrainer stiffness, as illustrated in Equation 11. For other cases, the iterative procedure in DesRoches and Fenves (2000) can be used.

RESTRAINER DESIGN PROCEDURE

The simplified restrainer design procedure is based on the empirical expression for the normalized restrainer stiffness presented in the previous section. The objective of the procedure is to provide sufficient restrainers to limit the displacement of the intermediate hinges in a multi-frame bridge to the allowable hinge seat, D_r. The mass, stiffness, and design ductility of the frames, and the restrainer properties are required for the procedure. The characteristics of the earthquake are represented by a pseudo-acceleration response spectrum, $S_a(T_{eff},\xi_{eff})$. Given this information, the simplified design procedure for intermediate hinge restrainers is as follows.

Step 1: Maximum Allowable Hinge Displacement

The first step is to determine the specified maximum hinge displacement, D_r, based on the available hinge seat, the bearing length and initial gap. A minimum bearing length of 51–102 mm (2–4 inches) is typically required (Caltrans 1990), depending on field conditions of the bridge. The target yield displacement for the restrainers is the difference between the maximum allowable hinge displacement and the restrainer slack, according to $D_y = D_r - D_s$, where D_y is the elongation at yield and D_s is the restrainer slack. The length of restrainers, L, is determined from $L = D_y E/F_y$, where F_y and E are the yield force and modulus of the typical cable restrainers, respectively.

Step 2: Initial Hinge Displacement

The initial hinge displacement is obtained from a response spectrum analysis using the CQC combination rule (Der Kiureghian 1980) as follows:

$$D_{eq_0} = \sqrt{D_{1_0}^2 + D_{2_0}^2 - \rho_{12} D_{1_0} D_{2_0}} \tag{12}$$

where D_{1_0} and D_{2_0} are the displacements of the individual frames, accounting for the effective period and damping as follows:

$$D_{i_0} = \frac{T_{\mathrm{eff}_i}^2}{4\pi^2} S_a(T_{\mathrm{eff}_i}, \xi_{\mathrm{eff}_i}) \tag{13}$$

where ρ_{12} is given by Equation 3, $T_{\mathrm{eff}} = \sqrt{W/K_{eff}\,g}$, and the effective damping, ξ_{eff}, based on the Takeda stiffness degrading model is (MacRae et al. 1994)

$$\xi_{\mathrm{eff}} = \xi + \frac{1 - \dfrac{0.95}{\sqrt{\mu}} - 0.05\sqrt{\mu}}{\pi} \tag{14}$$

Step 3: Calculate Number of Restrainers

The required restrainer stiffness is obtained from Equation 11. The number of restrainers is determined from $N_r = (K_r D_r)/(F_y A_r)$, where A_r is the area of the cable restrainer.

EXAMPLE OF SIMPLIFIED RESTRAINER DESIGN PROCEDURE FOR MULTIPLE-FRAME BRIDGES

As an example, the simplified restrainer design procedure is applied to a two-frame bridge located at two sites with different seismic hazards, San Francisco, CA, and Memphis, TN. The two frames have weights of 22.3 MN (5000 kips), and stiffnesses K_1

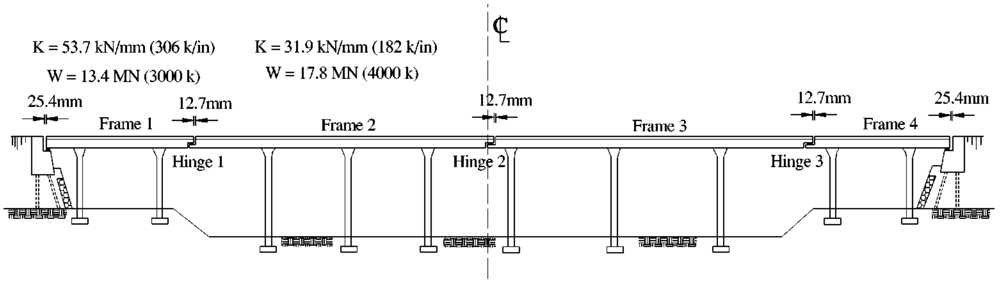

Figure 4. Typical multi-frame bridge used for comparison study.

$=357$ kN/mm (2040 kips/in) and $K_2=179$ kN/mm (1020 kips/in), respectively. The restrainers are designed for a seismic loading based on the NEHRP design spectrum for a return period earthquake of 475 years (10% probability of exceedance in 50 years). Both sites are assumed to be on stiff soil (Site Class D). The effective properties for the bridge are based on a displacement ductility of the bridge frames of $\mu=4$. Using the capacity spectrum method, a reduction factor is applied to the spectral displacements, based on the effective damping as follows (BSSC 1998):

$$B=\left(\frac{\xi_{\text{eff}}}{0.05}\right)^{0.3} \tag{15}$$

The bridge located in San Francisco has a hinge seat of 152 mm (6 in), with 63.5 mm (2.5 in) required for bearing, whereas the bridge in Memphis has an available hinge seat of 89 mm (3.5 in), with 51 mm (2 in) required for bearing. The cable restrainers used have a modulus of elasticity, $E=6.89(10)^4$ MPa (10,000 ksi), yield strength, $F_y=1.21(10)^3$ MPa (176.1 ksi), and cross-sectional area, $A=143$ mm^2 (0.22 in^2).

The results from the simplified procedure, shown in Table 1, give restrainer stiffnesses of 29.8 kN/mm (170 k/in) and 32.5 kN/mm (185 k/in) for Case 1 and Case 2, respectively. Although the design ground motion is larger for Case 1 compared with Case 2, the restrainer stiffness is larger for Case 2 because the allowable hinge displacement is much less for Case 2. Case 2 requires fewer restrainers than case 1 since the cables (2.13 m) are shorter. The multi-step procedure (DesRoches and Fenves 2000) is applied to the cases above to provide a comparison of the design procedures. The multi-step procedure results in stiffnesses of 35.1 kN/mm (200 k/in) and 36.0 kN/mm (205 k/in), respectively, for the two cases. The results from the simplified are within 15% of that from the multi-step procedure, without the need for modal analysis and iteration.

COMPARISON OF RESTRAINER DESIGN PROCEDURES

This section compares the equivalent static (Caltrans 1990) and AASHTO (1992) procedures, with the simplified design procedure for a typical multi-frame bridge, similar to that shown in Figure 4. The properties of the frames are taken from typical frames in the Northwest Connector at the Interstate 10/215 Interchange in Colton, CA, previously studied by the authors (DesRoches and Fenves 1997a). A gap of 25.4 mm (1.0 in) is assumed at the intermediate hinges and the abutments.

Table 1. Results of simplified procedure applied to multi-frame bridge

STEP	Case 1: SAN FRANCISCO, CA	Case 2: MEMPHIS, TN
1	***Allowable Hinge Displacement***	***Allowable Hinge Displacement***
	$D_r = 152 - 63.5 = 88.5$ mm (3.5 in.)	$D_r = 89 - 51 = 38$ mm (1.50 in.)
	$D_s = 12.7$ mm (0.50 in.)	$D_s = 12.7$ mm (0.50 in.)
	$D_y = 76.2$ mm (3.0 in)	$D_y = 25.4$ mm (1 in)
	$L = 76.2 * 6.89(10)^4 / 1.21(10)^3$	$L = 25.4 * 6.89(10)^4 / 1.21(10)^3$
	$= 4339$ mm (14.2 ft)	$= 1440$ mm (4.73 ft)
	Use 4.65 m (15 ft) cables	Use 1.52 m (5 ft) cables
2	***Relative Hinge Displacement***	***Relative Hinge Displacement***
	$T_{eff\,1} = 2\pi\sqrt{22.3/(9.81*357/4)} = 1.0$ sec	$T_{eff\,1} = 2\pi\sqrt{22.3/(9.81*357/4)} = 1.0$ sec
	$T_{eff\,2} = 2\pi\sqrt{22.3/(9.81*179/4)} = 1.4$ sec	$T_{eff\,2} = 2\pi\sqrt{22.3/(9.81*179/4)} = 1.4$ sec
	$\xi_{eff} = 0.05 + (1 - 0.95/\sqrt{4} - 0.05\sqrt{4})/\pi$	$\xi_{eff} = 0.05 + (1 - 0.95/\sqrt{4} - 0.05\sqrt{4})/\pi$
	$= 0.19$;	$= 0.19$;
	$B = (0.19/.05)^{0.3} = 1.5$	$B = (0.19/.05)^{0.3} = 1.5$
	From NEHRP map:	From NEHRP map:
	$S_1 = 0.60$, $F_v = 1.5$, $S_{D1} = 0.90$	$S_1 = 0.20$, $F_v = 2.0$, $S_{D1} = 0.40$
	$S_{a1} = 0.90g$, $S_{a2} = 0.64g$	$S_{a1} = 0.40g$, $S_{a2} = 0.29g$
	$\rho_{12} = 0.54$	$\rho_{12} = 0.54$
	$D_{10} = (0.9/1.5) * 9810/4\pi^2$	$D_{10} = (0.4/1.5) * 9810/4\pi^2$
	$= 149$ mm (5.86 in)	$= 66$ mm (2.60 in)
	$D_{20} = (0.64/1.5) * 9810 * 1.4^2/4\pi^2$	$D_{20} = (0.29/1.5) * 9810 * 1.4^2/4\pi^2$
	$= 208$ mm (8.19 in)	$= 94.2$ mm (3.71 in)
	$D_{eq0} = \sqrt{149^2 + 208^2 - 2*0.54*149*208}$	$D_{eq0} = \sqrt{66^2 + 94^2 - 2*0.54*66*94}$
	$= 179$ mm (7.05 in)	$= 80.6$ mm (3.17 in)
3	***Calculate Number of Restrainers***	***Calculate Number of Restrainers***
	$D_r / D_{eq0} = 88.5/179 = 0.50$	$D_r / D_{eq0} = 38/80.6 = 0.47$
	$K_r = 119/4 * (.50 + (.50 - .50^2)/.50)$	$K_r = 119/4 * (.50 + (.50 - .47^2)/.47)$
	$= 29.8$ kN/mm (170 k/in)	$= 32.5$ kN/mm (185 k/in)
	$N_r = 29.8 * 88.5/(1.21 * 143)$	$N_r = 32.5 * 38/(1.21 * 143)$
	$= 15.3$ restrainers (use 15 restrainers)	$= 7.1$ restrainers (use 8 restrainers)

To apply the simplified procedure, the individual frames are idealized as single-degree-of-freedom systems, with mass, m_i, and stiffness, k_i. The required parameters to solve for the restrainer stiffness are the modified frame stiffness, $K_m = K_1 K_2/(K_1 + K_2)$, the allowable hinge displacement, D_r, and the initial relative hinge displacement, D_{eq_0}. The maximum allowable hinge displacement, D_r, is selected based on the available hinge seat, the minimum bearing length, and initial gap. For this bridge, the available seat width is 119 mm (4.7 in). The target yield displacement of the restrainers is the difference between the maximum allowable hinge displacement and the restrainer slack, according to $D_y = D_r - D_s$, where D_y is the restrainer elongation at yield and D_s is the restrainer slack. The first and second vibration periods of the bridge are 1.0 second and 1.5 seconds, respectively. The bridge is subjected to the 1940 El Centro ground motion, and the 1995 Kobe City record, scaled to a peak ground acceleration of 0.70g. The in-

Table 2. Results of simplified procedure applied to multi-frame bridge

					1940 El Centro Ground Motion Record					
	Caltrans Procedure			AASHTO Procedure			Simplified Procedure			
Hinge	$K_r(N_r)$	D_{eq} (mm)		$K_r(N_r)$	D_{eq} (mm)		$K_r(N_r)$		D_{eq} (mm)	
		$\mu=1$	$\mu=4$		$\mu=1$	$\mu=4$	$\mu=1$	$\mu=4$	$\mu=1$	$\mu=4$
1	0 (**0**)	183	114	78.1 (**54**)	106	71	43.1 (**36**)	9.59 (**8**)	116	92
2	178 (**122**)	105	57	104 (**72**)	68	50	29.6 (**25**)	5.11 (**4**)	71	61
					1995 Kobe City Ground Motion Record					
	Caltrans Procedure			AASHTO Procedure			Simplified Procedure			
Hinge	$K_r(N_r)$	D_{eq} (mm)	$K_r(N_r)$	D_{eq} (mm)	$K_r(N_r)$	D_{eq} (mm)				
		$\mu=1$	$\mu=4$		$\mu=1$	$\mu=4$	$\mu=1$	$\mu=4$	$\mu=1$	$\mu=4$
1	0 (**0**)	318	114	78.1 (**54**)	102	81	71.1 (**49**)	8.75 (**6**)	102	104
2	73.3 (**50**)	140	51	104 (**72**)	69	90	48.3 (**33**)	4.01 (**3**)	71	69

D_r, Maximum allowable hinge displacement, 119 mm
K_r–Design restrainer stiffness (kN/mm)
N_r–Design number of restrainers (based on 20 ft restrainer cables)
D_{eq}–Relative hinge displacement determined from nonlinear time history analysis

teraction of the frames is accounted for by considering possible hinge conditions (completely open or completely closed) two frames away from the hinge being considered. Since the bridge is symmetric about the centerline, only hinges 1 and 2 are considered in the results.

Table 2 summarizes the results of restrainer design applied using the equivalent static, AASHTO, and simplified procedures. Using the simplified procedure, two cases are evaluated: a case with elastic frames and a case with yielding frames. While it is not realistic to expect elastic response for these levels of ground motion, the comparison is provided to show that ductile behavior of frames has a significant effect on the restrainer stiffness. For the yielding frames, the yield force is chosen such that the frames have an individual frame ductility demand of $\mu=4$. A nonlinear time history of the bridge is performed for each restrainer design method to determine the effectiveness of the method in limiting the response.

As illustrated in Table 2, the results of the equivalent static, AASHTO, and the simplified procedure vary considerably. For the elastic case with the 1940 El Centro ground motion the equivalent static procedure predicts zero and 122 restrainers for hinges 1 and 2, respectively. The simplified procedure predicts 36 and 25 restrainers for hinges 1 and 2, respectively. As shown in previous studies, the equivalent static procedure is typically unconservative for out-of-phase frames, and conservative for in-phase frames. The equivalent static procedure only considers the frame with the smallest displacement, and provides restrainers for that frame. Based on the equivalent static procedure, the displacement at hinge 1 is controlled by the displacement of frame 1. Including the stiffness of the abutments, the displacement of frame 1 was less then the allowable hinge displacement, and therefore did not require any restrainers. In reality, the displacement of hinge 1 is based on the relative displacement histories of frame 1 and frame 2. The more flexible frame 2 in this case contributed much more to the relative hinge displacement than frame 1. This is captured by the simplified procedure that predicts 36 restrainers for hinge 1. Comparing the relative hinge displacement determined by nonlinear time history analysis, D_{eq}, shows that the Caltrans design leads to a displacement at hinge 1 of $D_{eq}=183$ mm, exceeding the allowable hinge displacement by 64 mm. The simplified procedure resulted in displacements less than the allowable in both hinges. The AASHTO procedure, which is based only on the weight of the frame and the peak ground acceleration, provides a design that is adequate, albeit very conservative.

For the yielding frame case, both the equivalent static method and the AASHTO methods have the same design as the elastic case, since these procedures do not explicitly account for yielding frames. The simplified procedure predicts 8 and 4 restrainers for hinges 1 and 2, respectively. The considerable reduction of restrainers in the yielding case is due to two factors. First, the yielding frames have a much lower effective stiffness compared to the elastic cases, thus requiring much less force to restrain their displacements. Second, the hysteretic damping in the yielding frames promotes in-phase response, thereby further limiting the relative hinge displacement. Results of nonlinear time history analyses with the yielding frames shows that the restrainers provided by the simplified procedure are adequate in limiting the relative displacement.

The results for the 1995 Kobe City ground motion show similar trends to the 1940 El Centro ground motion. For the elastic case, the equivalent static procedure predicts zero and 50 restrainers for hinges 1 and 2, compared to 49 and 33 for the simplified procedure. The AASHTO procedure restrainer design remains the same since the peak ground acceleration is still 0.70g. Once again, a significant reduction in the number of restrainers is obtained in going from elastic frames to yielding frames.

CONCLUSIONS

A simplified design procedure for restrainers at intermediate hinges of multi-frame bridges has been presented. Currently, most restrainer design procedures require iterations to converge to a design (DesRoches and Fenves 2000, Trochalakis et al. 1997, Caltrans 1990). The simplified procedure allows for the calculation of the required restrainer stiffness to limit hinge displacement, without iteration. The procedure is based on an empirical relationship for the restrainer stiffness as a function of the frame stiffnesses, initial hinge displacement, target displacement, and design ductility. The inelastic behavior of the frames is represented by the substitute structure method. The procedure accounts for the dynamic characteristics and out-of-phase motion of the frames through a modal expression for the relative hinge displacement using the effective properties of the frames. Based on the range of parameters studied, the procedure is effective for bridge frames with period ratios $T_1/T_2 \geqslant 0.60$. The procedure does not account for the effects of abutments in the response of the hinge. Therefore, it should only be used for hinges located at least one frame from the end frame. However, if the end frames are very stiff compared with the interior frames, the procedure may be valid for the intermediate hinges closest to the abutments.

Parameter and case studies show that the simplified procedure provides restrainer stiffnesses that compare well with nonlinear time history analysis. The ability of the procedure to account for the inelastic response of the frames and the dynamic response of the frames results in restrainer designs that require fewer restrainers than are determined by current procedures.

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APPENDIX A: SYMBOLS LIST

A	Area of Restrainer Cable
B	Effective Damping Reduction Factor
$\underline{D}$	Modified Frame Displacement Ratio
D_{i_0}	Individual Frame Displacement
D_{ave}	Average Independent Frame Displacement
D_{eq}	Relative Hinge Displacement
D_{eq_0}	Unrestrained Relative Hinge Displacement
D_i	Modal Hinge Displacements
D_r	Specified Allowable Hinge Displacement
D_s	Restrainer Slack
D_y	Restrainer Elongation at Yield
E	Modulus of Elasticity
F_y	Yield Force of Cable Restrainer
$\underline{K}$	Normalized Restrainer Stiffness
K_i	Frame Stiffnesses
K_m	Composite Frame Stiffness
K_r	Restrainer Stiffness
L	Length of Restrainer Cable
N	Number of Restrainers
T_{eff_i}	Effective Period of Frame
$\underline{T}_g$	Effective Period of Ground Motion
T_g	Ground Motion Period Ratio
T_i	Frame Periods
T_L	Longer Period of Independent Frame

T_S	Shorter Period of Independent Frame
S_a	Pseudo-Acceleration Response Spectrum
W	Weight of Bridge Frame
β	Frame Frequency Ratio
μ	Frame Ductility
r_{12}	Cross Correlation Coefficient
ξ_i	Modal Damping Ratios
ξ_{eff_i}	Effective Modal Damping Ratios

APPENDIX B

Table B1 lists the ground motion records used in the analysis.

Table B1. List of ground motion records used in analysis

Earthquake record	Location	Mag M_S	EPD[a] (km)	PGA[b] (g)
1940 Imperial Valley	El Centro	6.9	12	0.35
1971 San Fernando	Pacoima Dam	7.4	8	1.36
1979 El Centro	Bonds Corner	6.6	28	0.78
1980 Mammoth Lk.	HS Gym	6.5	11	0.34
1984 Morgan Hill	Coyote Dam	6.2	24	1.12
1987 Whittier	Alhambra	6.1	7	0.25
1989 Loma Prieta	Saratoga	7.1	28	0.47
1989 Loma Prieta	Corralitos	7.1	8	0.65
1992 Cape Mend.	Fortuna	6.9	28	0.12
1992 Cape Mend.	Petrolia	6.9	5	0.70
1992 Landers	Amboy	7.5	74	0.15
1992 Landers	Baker Fire	7.5	122	0.11
1994 Northridge	Sylmar	6.7	15	0.90
1994 Northridge	Arleta	6.7	10	0.32
1994 Northridge	Pico	6.7	31	0.19
1994 Northridge	Pacific Dam (KC)	6.7	18	0.52
1994 Northridge	LA Obrego Park	6.7	39	0.45
1994 Northridge	Downey County	6.7	47	0.25
1994 Northridge	Tarzana	6.7	5	0.65
1994 Northridge	Inglewood	6.7	42	0.26
1994 Northridge	Pacific Dam (DS)	6.7	17	0.50
1994 Northridge	Mt. Wilson	6.7	45	0.26
1994 Northridge	Lake Hughes	6.7	44	0.27
1995 Kobe	Osaka	6.9	17	0.08
1995 Kobe	Fukushima	6.9	17	0.04
1995 Kobe	Kobe	6.9	5	0.85

a. Epicentral distance.
b. Peak ground acceleration.

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