SIMULATED MICROMECHANICAL MODELS USING ARTIFICIAL NEURAL NETWORKS

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ABSTRACT: A new method, termed simulated micromechanical models using artificial neural networks (MMANN), is proposed to generate micromechanical material models for nonlinear and damage behavior of heterogeneous materials. Artificial neural networks (ANN) are trained with results from detailed nonlinear finite-element (FE) analyses of a repeating unit cell (UC), with and without induced damage, e.g., voids or cracks between the fiber and matrix phases. The FE simulations are used to form the effective stress-strain response for a unit cell with different geometry and damage parameters. The FE analyses are performed for a relatively small number of applied strain paths and damage parameters. It is shown that MMANN material models of this type exhibit many interesting features, including different tension and compression response, that are usually difficult to model by conventional micromechanical approaches. MMANN material models can be easily applied in a displacement-based FE for nonlinear analysis of composite structures. Application examples are shown where micromodels are generated to represent the homogenized nonlinear multiaxial response of a unidirectional composite with and without damage. In the case of analysis with damage growth, thermodynamics with irreversible processes (TIP) is used to derive the response of an equivalent homogenized damage medium with evolution equations for damage. The proposed damage formulation incorporates the generalizations generated by the MMANN method for stresses and other possible responses from analysis results of unit cells with fixed levels of damage.

INTRODUCTION

The process of damage development and failure in composite materials is very complicated. Although the equivalent elastic behavior of the material and some nonlinear response may be considered to depend on the average stress or strain in the phases, equivalent nonlinear response caused by damage is strongly dependent on local geometric and material details. Therefore, any analytical micromechanical damage analysis should properly take into account the detailed microstructure, spatial deformation fields, existing defects, criteria for local failure and its evolution, and the way different defects and modes of failure interact as loading progresses. Modeling all these aspects is difficult, if not impossible. However, it is doubtful that overall structural response is sensitive to all these variables. Therefore, it is not surprising to find that most of the current micromechanical failure and damage theories are formulated using average deformation quantities and in the context of continuum damage and mechanics of materials (Pecknold and Haj-Ali 1993; Haj-Ali and Pecknold 1996).

The difficulty of obtaining exact analytical models for fiber composites, especially when the internal geometry and inelastic matrix response are explicitly recognized, naturally led researchers to use numerical and approximate methods. Finite-element and finite-difference methods can be used to derive the effective properties of composite materials, where the internal geometry and interface conditions can be explicitly modeled. When using numerical methods to generate the effective response of a composite material, a repeating unit cell must be constructed. Therefore, periodic symmetry conditions must exist in the representative volume element (RVE) of the material. Various fiber-packing geometries can be considered, such as the square, rectangular, and hexagonal fiber arrays. Among these, only the hexagonal array yields a transversely isotropic elastic effective moduli for fiber reinforced composites.

A numerical finite-difference method was used by Tsai et al. (1966a,b) to generate the effective response of unidirectional composites. Adams (1970) and Foye (1973) used the finite-element method for composites, with elastic-plastic constitutive relations for the matrix elements. Dvorak (1991) and Bahei-El-Din and Dvorak (1989) have extensively studied the elastic-plastic response of metal matrix composites using the finite-element method and used the periodical hexagonal array (PHA) as the fiber-packing geometry.

The Mori-Tanaka micromechanics method was developed to determine the average stresses in a matrix including precipitates with eigenstrains (Mori and Tanaka 1973). It has also been used to determine the effective moduli of composite materials. Benveniste (1987) gave a complete simplified formulation of this method for composites. The effective properties of fiber composites with transversely isotropic phases was given by Dvorak (1991). Gavazzi and Lagoudas (1990) and Lagoudas et al. (1991) used the Mori-Tanaka averaging scheme for modeling the elastic-plastic response of metal matrix composites. They computed the Eshelby tensor using the instantaneous matrix material properties. Also, they approximated the form of the concentration tensors by making the instantaneous moduli a function of the average stresses in the matrix phase, and used the backward difference to integrate the incremental elastic-plastic equations. The method was compared to experimental and the (PHA) finite-element hexagonal unit cell for both fibrous and particulate composites. Good results were obtained for particulate composites with small concentration of particles. However, in the case of fibrous composites, the Mori-Tanaka method underestimates the plastic strains under certain transverse loading paths.

The method of cells (MC) was derived (Aboudi 1982, 1991) for inelastic analysis of composite materials. Numerous studies by Aboudi and his colleagues have dealt with the application and verification of the MC for metal and resin-based composite materials. In the MC, the medium is idealized with fibers that have rectangular cross sections arranged in a doubly periodic array. A representative unit cell is constructed for the

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medium and divided into four subcells. An expansion of the displacement vector in Legendre polynomials is made in terms of the local coordinate systems located at the center of each subcell. Displacement and traction continuity along with periodic boundary conditions are used to set up a system of equations to solve for the expansion coefficients. The displacement and traction continuity are satisfied only in an average sense. Currently, the most widely applied version of the method of cells is the four subcell model with linear displacement expansion. Paley and Aboudi (1992) developed the generalized method of cells (GMC). In this generalization, the unit cell is divided into multi-subcells, and the linear displacement expansion form is used for all the subcells.

The finite-element method is also used to compare the performance and response prediction of different analytical micromodels of composite materials. Lissenden and Herakovich (1992) and Noor and Shah (1993) compared different micromodels with results from the FE method for the case of linear thermoelastic effective properties. Suresh and Brockenbrough (1994) performed a similar comparison study for the case of inelastic response. Christensen (1990) compared the generalized self-consistent scheme to two other approximate methods, the differential scheme and the Mori-Tanaka methods.

The ability of the above micromechanical methods to accurately predict the overall nonlinear material response is limited in the presence of damage and localization. This is due to their inherent limitation. The formulation is carried out in terms of average state variables, and it often lacks accurate spatial characterization of deformations in the phases. Therefore, detailed numerical methods in micromechanical analysis of heterogeneous materials are crucial in order to investigate the effective response, especially in the presence of inelastic and local effects, such as inelastic material behavior, interface debonding, and crack propagation. It is difficult, however, to use detailed numerical models in a multiscale global-local type analysis of composite structures. This is due to the large computational efforts involved and the different boundary conditions that are needed in order to generate the effective response for a general multiaxial loading state. Having said that, Feyel and Chaboche (2000) have recently proposed such an approach, using an interleaved multilevel finite element that explicitly models and recognizes the behavior of the microstructure during the global-scale nonlinear structural analysis. Costanzo et al. (1996) developed a homogenization scheme to derive the effective response of an elastoplastic composite system with growing damage. Caiazzo and Costanzo (2000a,b) have developed a computational method, termed discrete damage space homogenization method (DDSHM), where approximations are generated for the energy release rate (ERR) of an RVE for a layered composite with growing cracks. Equivalent continuum response is then generated using thermodynamics of irreversible processes (TIP). Haj-Ali et al. (1998, 1999) also proposed a similar method, where detailed nonlinear finite-element (FE) analysis of a periodic heterogeneous microstructure is carried out discretely and separately with and without fixed levels of damage. These results are then used to train an artificial neural network (ANN) in order to generate a continuous multiaxial nonlinear response of the homogenized continuum. The pretrained ANN is then used as a material model within a general displacement-based FE for the nonlinear analysis of composite structures. This paper summaries and extends the micromechanical models using artificial neural networks (MMANN) method by generating equivalent damaged continuum formulation with growing damage using TIP in a parallel approach to the work of Caiazzo and Costanzo (2000a,b).

ANN is composed of computational cells (neurons) that are connected with weighted links. A forward-pass type network consists of a small number of layers with several cells in each. A cell in a layer can only communicate (provide input) with the cells in the next layer. The first and last layers include the input and output cells. Mathematical optimization algorithm is usually used to adjust the connection strengths (weights) between the cells during the training process. This is a minimization process where the objective function is the cumulative error for a given data set of associated input and output vectors. The optimization variables are the internal weights.

ANN can be used to associate and identify complex nonlinear relationships between variables in a given data set. ANN can be viewed as a mathematical approximation of a vector function (output) where its components are functions of the same variables (input). The domain of approximation is directly related to the number of neurons and layers (ANN architecture). These are often not specified a priori and can be determined during training along with the connection weights. Knowledge representation can be impeded with different ANN models (Hertz et al. 1991; Ritter et al. 1991). ANN is an effective tool when relationships and approximation of variables are needed from a given large volume of data. They are robust, noise tolerant, and are capable of generalizing (Ghaboussi et al. 1998).

ANN has been applied to generate nonlinear constitutive models for different materials (Ghaboussi et al. 1991, 1994, 1998; Pidaparti and Palakal 1993; Ellis et al. 1995; Haj-Ali et al. 1998, 2001). A new indirect method, called autoprogressive training, for training neural network material models from structural tests, was recently proposed by Ghaboussi et al. (1998). Structural test responses, for example, load versus deflection data, are used to train the neural network to learn the nonlinear response and behavior of the material. A fundamental premise of this method is that structural tests usually generate a large number of spatial patterns of stresses and strains that can be used for training.

This work deals with a new complementary method to autoprogressive training, termed MMANN. In this method, micromechanical material models are generated from simulated nonlinear FE models of a unit cell for a periodic heterogeneous medium. The response of the heterogeneous material can be simulated with and without damage using detailed FE models. The trained MMANN models are used to capture the multiaxial effective material response. This class of constitutive models can be easily integrated within a nonlinear FE for the nonlinear analysis of composite structures.

IMPLEMENTATION OF MMANN MATERIAL MODELS

A proposed hierarchical framework for nonlinear analysis of composite structures using MMANN material models is illustrated in Fig. 1. It describes a global-local nonlinear structural approach for analysis of thin layered structures. A pretrained MMANN material model is used to generate the effective stress response for a single unidirectional lamina in its local material system. Three layers with different orientations are depicted in order to illustrate the fact that only one MMANN model is needed to generate their stress response in their material orientation systems. Standard stress and strain transformations are used before and after evaluating the effective stress from the MMANN model. The classes of finite-element models that can be used with this framework are continuum plane stress, plate, and shell elements. Therefore, each lamina is assumed to be under plane-stress conditions. The strain provided at each integration point is applied to each lamina in its global coordinate system in order to derive the laminate stress resultants. This is done while performing through-thickness integration of the stresses in the laminae.

The MMANN model is used in a simple forward-substitution calculation and requires minimal computational effort

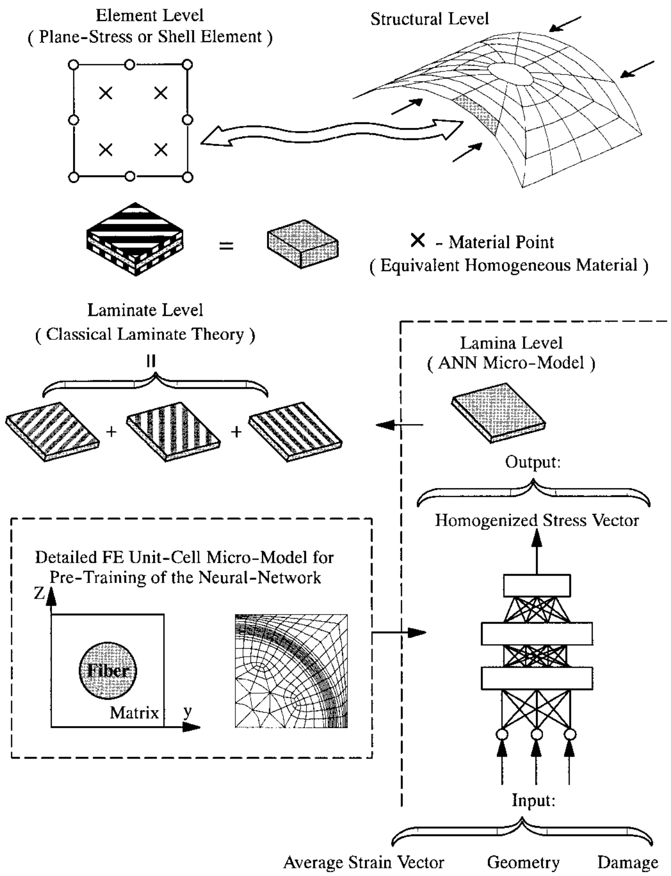

FIG. 1. Integrating Pretrained MMANN Material Models in Nonlinear and Damage Analysis of Composite Structures

once the pretraining for the MMANN is achieved. Therefore, the computational premium of the MMANN method is mainly due to the FE unit-cell analyses and during the training of the ANN. Although this computational effort may be substantial, it is negligible compared with the computational efforts consumed using standard inelastic and damage models that are repeatedly performed at the Gaussian integration points of a typical structural analysis.

A nonlinear plane-stress constitutive model for unidirectional composites is trained using results from detailed nonlinear FE analyses of a repeating unit cell, with and without induced damage modes, e.g., voids and matrix or interface cracks. The analyses are carried out over a preselected and relatively small number of applied average-strain paths. The FE results are used to construct a discrete parametric data set to train the ANN. This set is usually in the form of equivalent stress-strain response for a range of selected geometric and damages parameters. The powerful capability of ANNs to generalize a given set of data is then used in order to train a parametric approximate constitutive damage mode that can be used in the nonlinear analysis of composite structures. This method allows the introduction of local variables and complex behavior that are not usually considered in simplified micromechanical models.

NONLINEAR FRP MMANN MODEL

The first application presents a MMANN nonlinear model for a unidirectional fiber reinforced plastic (FRP) material. The goal is to demonstrate the training and prediction capability of the proposed MMANN model for a unidirectional lamina. The nonlinear response of the matrix material is included in the unit cell (UC) FE models. The medium is subjected to combined axial, transverse, and longitudinal-shear loading.

The stress and strain vector space for a unit cell (training domain) can be visualized as a 3D space with (ε_{11}, ε_{22}, γ_{12}) as its Cartesian axes. Angles θ and α are used to define the strain paths for a constant magnitude strain vector, using a spherical coordinate system (R, θ, α). These strain components are defined in the local material coordinate of a unidirectional composite material where the first Cartesian coordinate is aligned with the fiber direction. Detailed FE analyses are performed for a periodic unit cell made of AS4 graphite fiber with 3501-6 epoxy matrix. A unit-cell FE model is constructed for fiber volume fraction (FVF) of 0.405, similar to the mesh shown in Fig. 2 (FVF = 0.6). The nonlinear response of the matrix constituent is included in the FE models using deformation plasticity; whereas, the behavior of the fiber material is modeled as elastic and transversely isotropic. The material properties are listed in Table 1. This table includes the elastic properties of the fiber and the matrix constituents along with the Ramberg-Osgood nonlinear shear stress-strain parameters. The maximum magnitude of the strain vector is limited to 3.0%, with 30 strain increments along each path. The training strain-path orientations include the angles: $\theta = n\pi/8$, (n = 0, 1, 2, . . . , 15) and $\alpha = m\pi/8$, (m = 0, 1, 2, 3).

The MMANN architecture is depicted in Fig. 3 for the case of total strain formulation. Incremental constitutive formulation can be performed. Fig. 4 demonstrates an alternative ANN for generating incremental constitutive models. This scheme is denoted as the three-point (3-pt) scheme where the ANN's input and output include previous stress and strain points in order to account for history dependent behavior. The assumption is that the ANN will generalize these complicated relations from the training cases. However, the current case is a simple one. It does not include damage and includes only

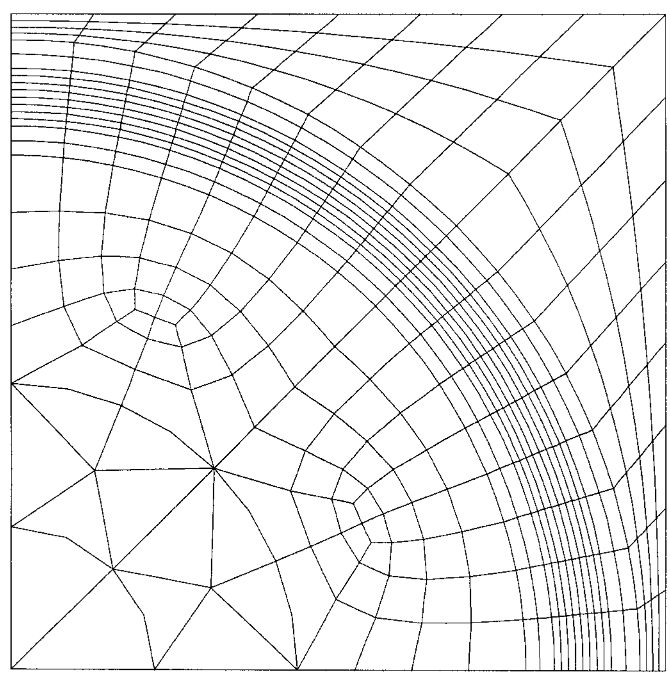

FIG. 2. Finite Element Mesh for Quarter Unit-Cell Model with Nonlinear Undamaged (Perfect-Bond) Medium (FVF = 0.6)

TABLE 1. Properties of AS4 Graphite Fiber and 3501-6 Epoxy Matrix

	E_1 (10^3 ksi)	E_2 (10^3 ksi)	G_{12} (10^3 ksi)	ν_{12}	ν_{23}	β	τ_o (ksi)	n
AS4 Graphite	27	2.5	5	0.3	0.25	$\gamma = \frac{\tau}{G} + \beta \frac{\tau_0}{G}\left(\frac{\tau}{\tau_0}\right)^n$		
3501 Epoxy	0.73			0.4		1.0	15	6

monotonic loading with total stress-strain for a fixed UC geometry (Cf = 0.405, packing ratio = 1). A set of ANN connection weights is generated as a result of successful training. The next step involves verification and evaluation of the ANN; i.e., has the new ANN micromodel adequately generalized the behavior characteristics of the material? This is determined by examining the ANN's ability to predict the unit cell effective

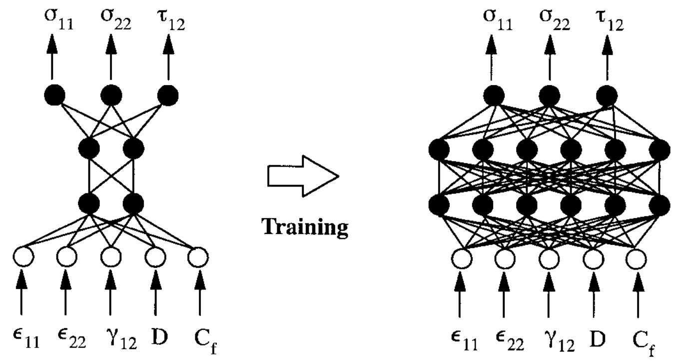

FIG. 3. Schematic Representation of MMANN Model Where Input for ANN Includes Variables in Form of Total Inplane Strain, Damage, and FVF; Output Is Inplane Stress

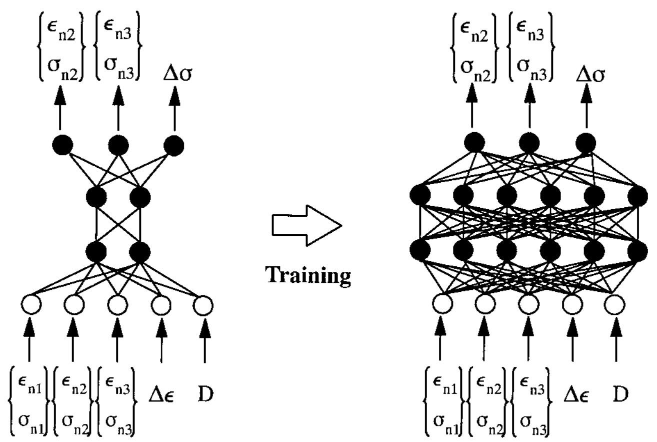

FIG. 4. Schematic Representation of MMANN Model for Incremental Model Using 3-pt Scheme

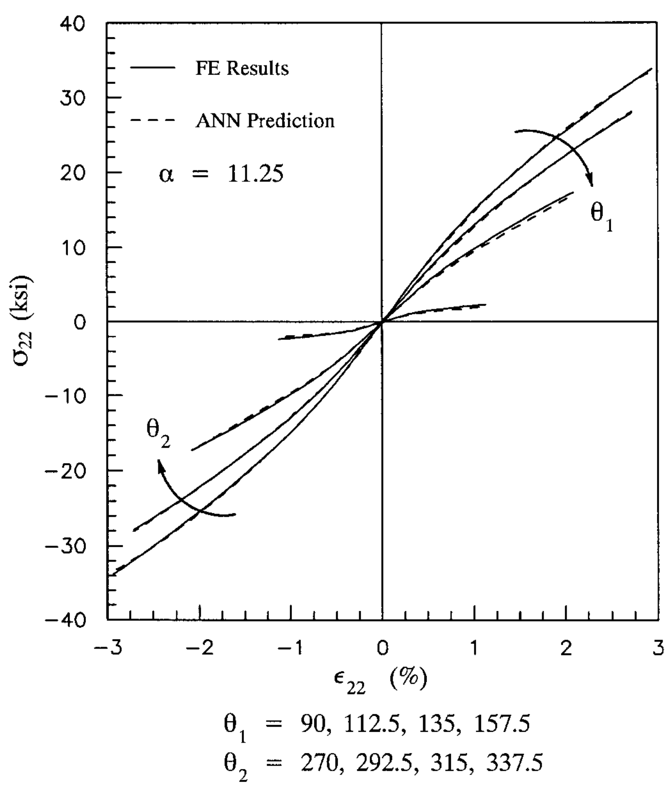

FIG. 5. ANN Micromodel Prediction of Effective Transverse Response with Different Applied Multiaxial Strain Paths That Are Defined by Angles α, θ_1, and θ_2

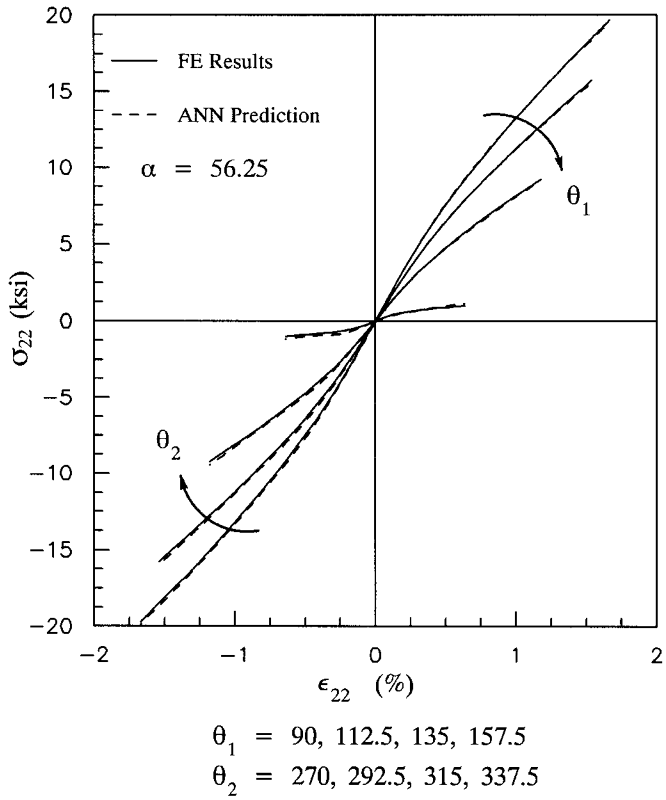

FIG. 6. ANN Micromodel Prediction of Effective Transverse Response with Different Applied Multiaxial Strain Paths That Are Defined by Angles α, θ_1, and θ_2

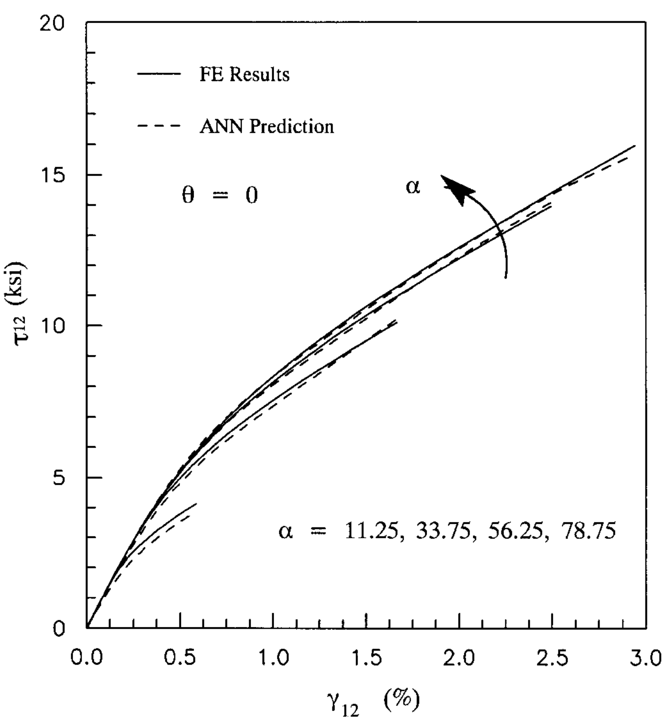

FIG. 7. ANN Micromodel Prediction for Effective Axial-Shear Stress with Applied Axial and Transverse Strain Loading

behavior for strain paths that are *not included in the training cases*.

The prediction of the ANN for transverse tension and compressive behavior is compared with additional FE analyses, not part of the training data, and is shown by dashed lines in Figs. 5 and 6. It is seen that the transverse response that was predicted by the MMANN model is in excellent agreement with the FE results for these cases. The FE unit-cell model yields similar responses for compression and tension loading in both

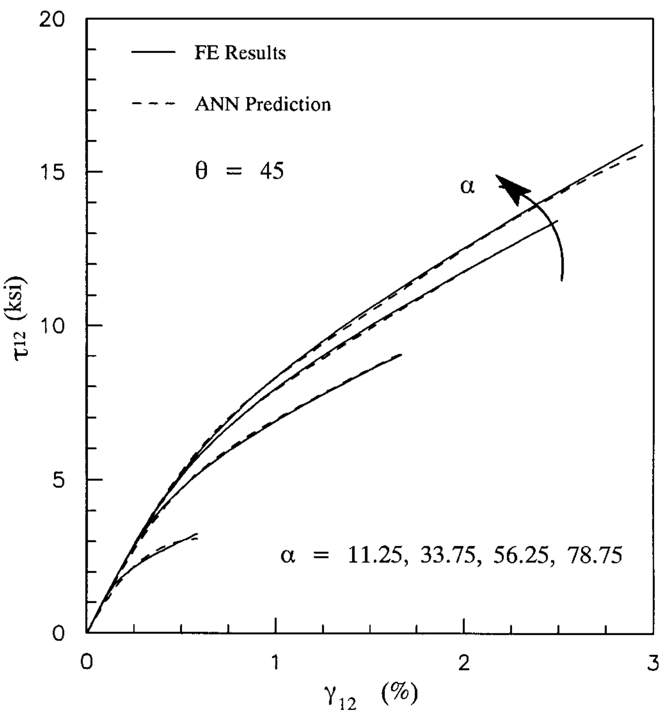

FIG. 8. ANN Micromodel Prediction for Effective Axial-Shear Stress with Applied Axial and Transverse Strain Loading

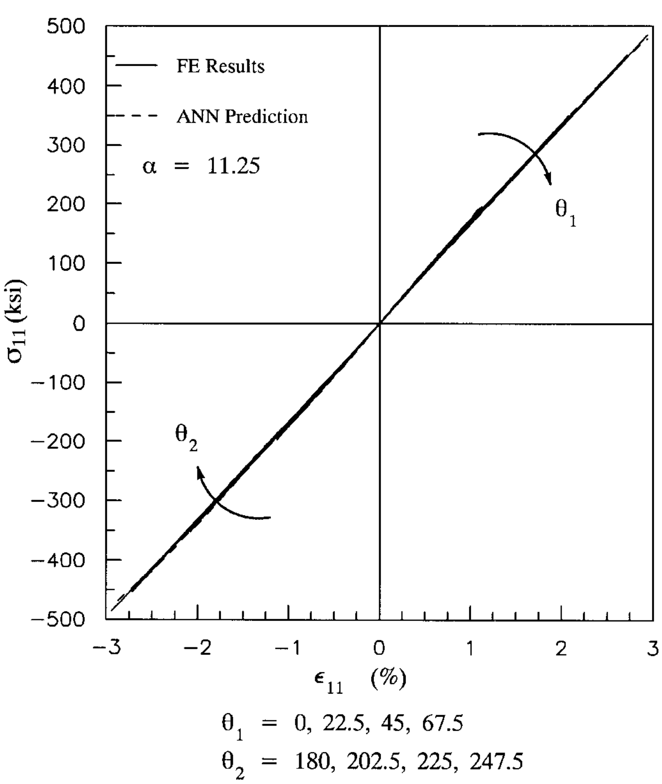

FIG. 9. ANN Micromodel Prediction of Effective Axial Stress with Applied Multiaxial Strain Paths That Are Defined by Angles α, θ_1, and θ_2

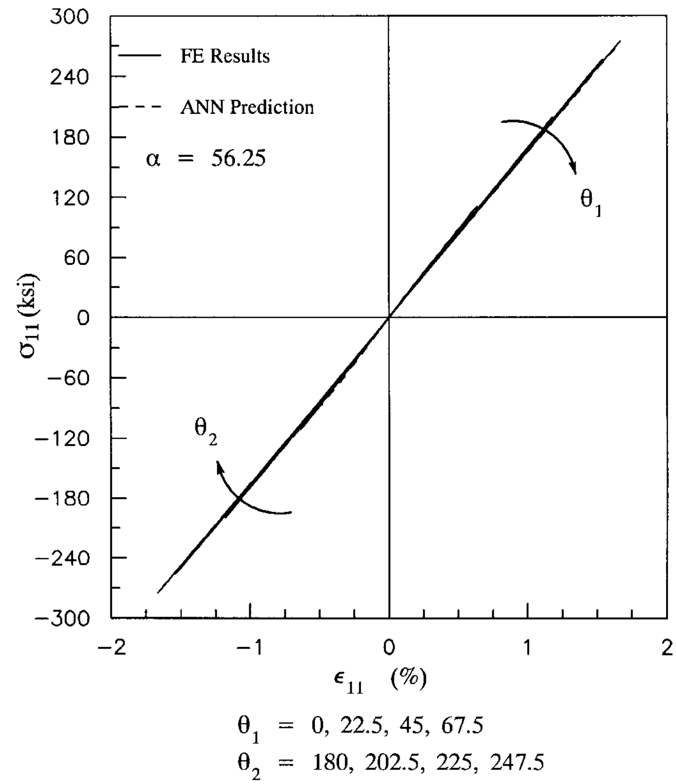

FIG. 10. ANN Micromodel Prediction of Effective Axial Stress with Applied Multiaxial Strain Paths That Are Defined by Angles α, θ_1, and θ_2

axial and transverse modes. This is due to the perfect bond that is assumed to exist between the fiber and the matrix and because of the use of deformation plasticity for the matrix constituent. The MMANN model is able to predict this trend without bias and a priori assumptions. Figs. 7 and 8 demonstrate the ability of the micromodel to predict axial-shear behavior when combined with applied transverse and axial modes. Again, excellent agreement is exhibited in the ability to predict the actual behavior of the detailed FE model. Similar response predictions for the axial (fiber) mode response of the MMANN model are shown in Figs. 9 and 10.

MMANN DAMAGE MODEL

The second MMANN model is generated using results from detailed FE analyses of a periodic unit cell made of boron/aluminum metal-matrix composite (MMC). The nonlinear response of the aluminum matrix is included in the FE models. The induced damage is in the form of fiber/matrix interface cracks, where the medium is subject to combined longitudinal shearing and transverse loading. The FE results are used to train the ANN, in which the crack angle is used as a damage parameter, along with the fiber-volume fraction as an internal geometric variable. Good prediction by the trained ANN is demonstrated for stress-strain response of a unit cell, with crack angles and fiber-volume fractions that are not part of the training set.

The medium is assumed to be periodic in order to allow adopting a unit cell as the RVE, as shown in Fig. 11. The nonlinear response of the aluminum matrix is included in the FE models using incremental plasticity with isotropic hardening. The behavior of the fiber material is modeled as elastic and isotropic, as shown in Table 2. The imperfect bond between the fiber and matrix is assumed to be in the form of two symmetrically located interface cracks. FE results are generated for a UC, with and without induced damage at the fiber/matrix interface, under combined longitudinal shearing and

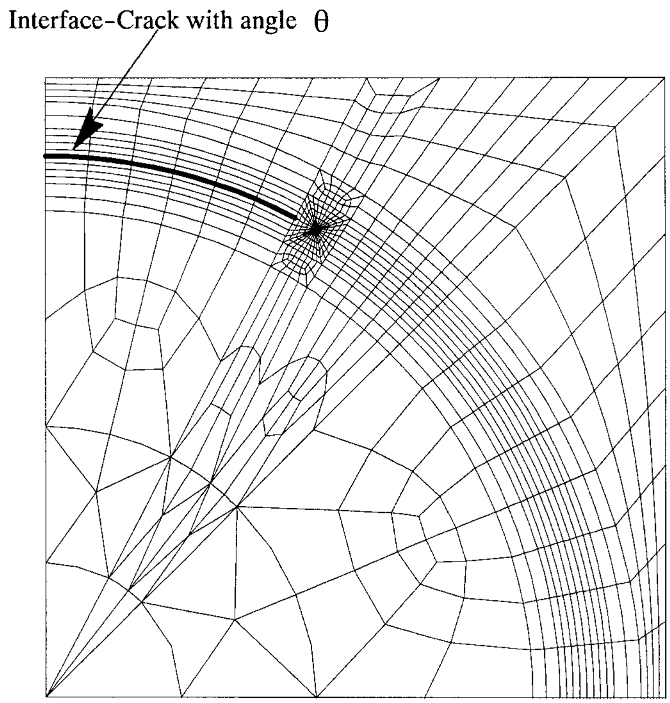

FIG. 11. Finite Element Mesh for Quarter Unit-Cell Model for Damaged Medium (Interface Crack)

TABLE 2. Properties of Boron Fiber Aluminum Matrix

	E_1	E_2 (msi)	G_{12}	ν_{12}	ν_{23}	β	τ_o	n
Boron	58			0.2		$\gamma = \frac{\tau}{G} + \beta \frac{\tau_0}{G}\left(\frac{\tau}{\tau_0}\right)^n$		
Aluminum	8.69			0.25		1.0	6	5

transverse strain paths. This data is used to train the ANN with the crack angle as a damage parameter and the fiber-volume fraction as an internal geometric variable. Figs. 2 and 11 show the FE mesh for undamaged and damaged unit cells (FVF = 0.6), respectively. Detailed mesh is introduced at the crack tip in order to accurately model the stress singularity and prevent typical oscillations in a bimaterial interface crack. A mesh convergence study was performed to justify the proposed mesh. It is found that the used mesh is more than adequate to generate the average stress/strain.

Initially, a fiber volume fraction of 0.6 is considered with crack angles of θ = 0°, 10°, 20°, 30°. Nine applied remote strain paths are chosen for training. They are denoted by T_i, where $i = 1, \ldots, 9$. Results from two additional strain paths, denoted by V_1 and V_2, are not employed in the training; these are used to verify the ability of the trained MMANN to predict the response for cases that are not part of the training set. Analyses were usually terminated when either the equivalent axial-shear strains or transverse strains reached a 1% magnitude. It is assumed that this is a large enough magnitude to cause a structural type global damage mode. This behavior is considered beyond the scope of the present study.

The next step examines the prediction capability of the MMANN model for this type of damaged MMC system. The model's ability to predict MMC behavior for strain paths and crack angles that are not part of the training cases is examined. Additional detailed FE analyses of the RVE are performed for new cases of strain paths combined with different crack angles, $V1$ and $V2$. A new combination of crack angles, θ = 0°, 5°, 15°, 25° are chosen with these patterns. The strain paths for training cases $T1$ to $T9$ are also used along with a new crack angle, θ = 15°, as additional verification cases.

The prediction of the ANN for transverse tensile and compressive behavior, strain paths $V1$ and $V2$, is shown in Fig. 12. It is interesting to note that in all these cases neither the strain paths nor the crack angles are part of the training set. The ANN results (dashed lines in Fig. 12) exhibit a remarkable performance in capturing the actual behavior of the MMC composite, for both tensile and compressive loading, along with a wide range of interface cracks. Fig. 13 demonstrates the ability of the MMANN model to predict the axial-shear behavior when combined with an applied transverse tensile mode, loading pattern $V1$. Again, excellent agreement is exhibited in the ability of the MMANN to predict the actual behavior of the material. Fig. 14 shows the same behavior as shown in Fig. 13 but with a combined compressive transverse loading, pattern $V2$. It is interesting to note that the FE results did not completely converge with the provided tight tolerance for the case of θ = 15°. This is due to the compression contact that exists at the crack faces for loading pattern $V2$. This con-

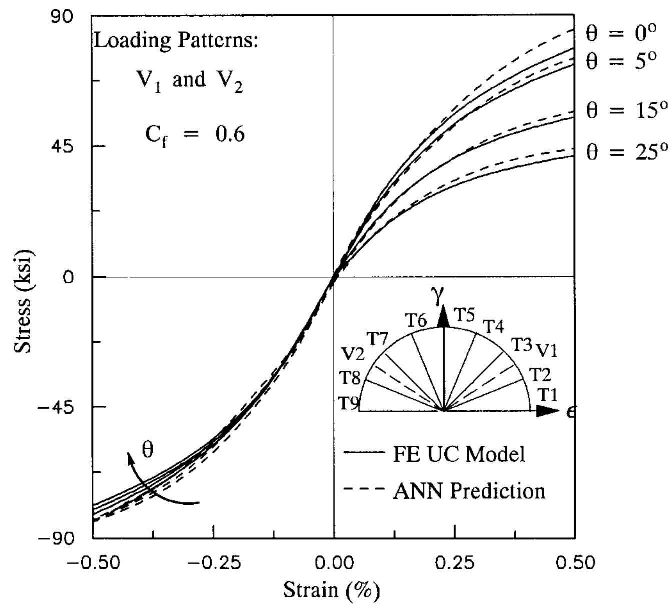

FIG. 12. Prediction of ANN Micromodel for Effective Transverse Stress with Different Crack Angles

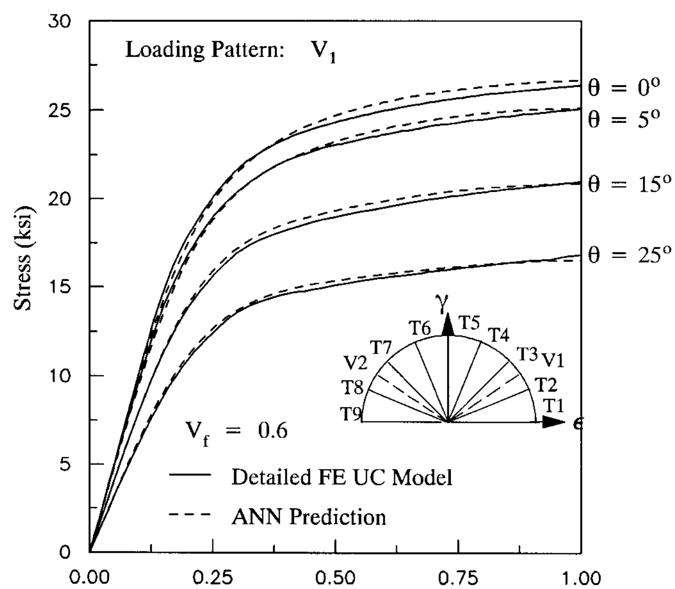

FIG. 13. Prediction of ANN Micromodel for Effective Axial-Shear Stress with Different Crack Angles and Applied Strain Path

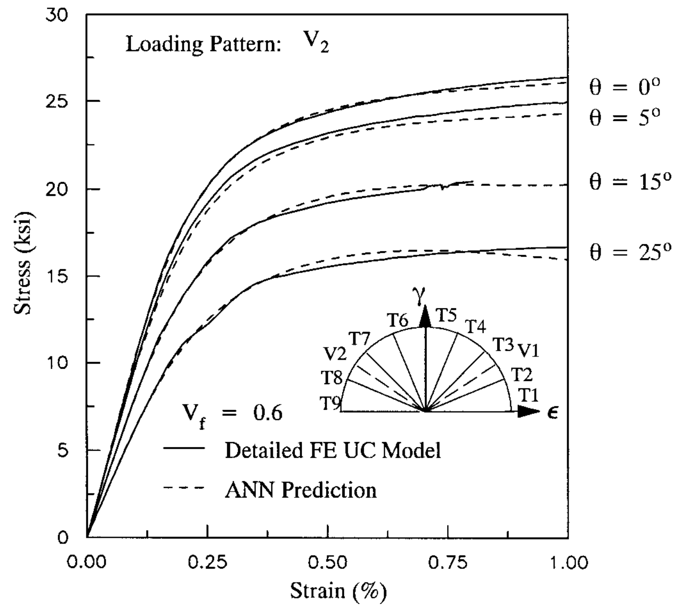

FIG. 14. Prediction of ANN Micromodel for Effective Axial-Shear Stress with Different Crack Angles and Applied Strain Path

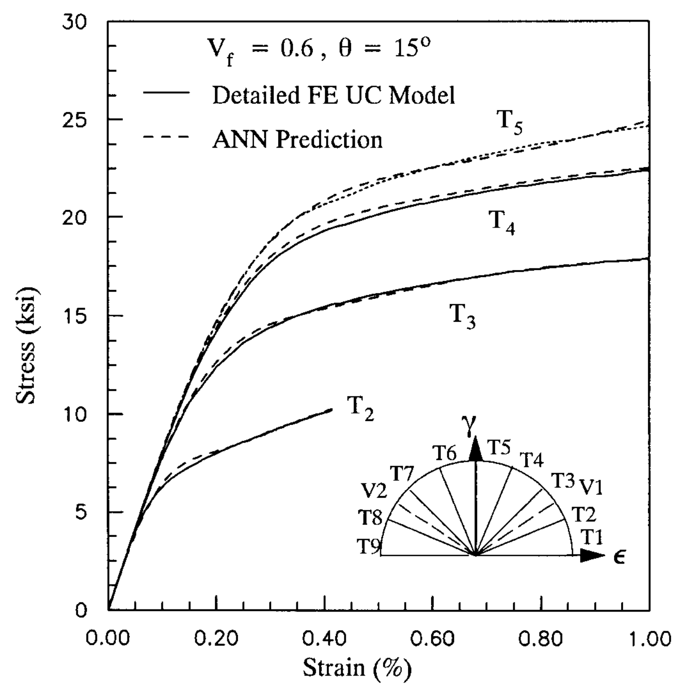

FIG. 15. Prediction of ANN Micromodel for Effective Axial-Shear Stress with Different Combined Tensile Loading

vergence can be fixed by relaxing the convergence tolerance or adding more intermediate increments. Fig. 15 illustrates the axial-shear response of the ANN model for the pretrained tensile strain paths *T*2 to *T*5, with an interface crack of $\theta = 15°$ that is not part of the training. Overall, the predicted results from the MMANN model is in excellent agreement with the detailed FE models.

It is clear from the FE results that the effective behavior of the RVE is different under compressive or tensile transverse modes. This is due to crack opening or closing mechanisms that occur in these modes. Also, axial shear is strongly dependent on the combined transverse loading. On the other hand, it is seen that the transverse loading is far less sensitive to combined axial shear. The interface crack length or angle influences both tensile-transverse loading and axial-shear loading. These different observed modes of nonlinear behavior can only add to the motivation of this study in using MMANN for modeling complex material response.

EFFECTIVE MEDIUM WITH DAMAGE GROWTH

In previous sections, the MMANN method is used to generate the homogenized effective stress of a unit cell with fixed levels of damage variables. The objective of this section is to use the generated approximations from MMANN and employ TIP to describe the response of an equivalent homogenized damage medium with damage evolution. This process is needed in order to integrate the proposed MMANN models within a nonlinear structural analysis framework.

In previous sections, detailed FE unit-cells models are generated and the MMANN method was used to approximate the effective stress as a function of the applied average strain, geometry, and damage variables

$$\hat{\sigma}_{ij} = \hat{\sigma}_{ij}(\varepsilon^e_{ij}, D_n) \quad (1)$$

where ε^e_{ij} is the average elastic strain specified through the applied displacements on the boundary, and D is the damage vector, with $n = 1 \ldots N$ damage variables. The symbol $\hat{\sigma}$ is used to denote an approximate relation that is generated using the ANN model.

The free energy is decomposed in an isothermal process, in terms of the external state variables, into elastic and inelastic parts

$$\psi = \frac{1}{\rho_o}\bar{\psi} = \psi^e(\varepsilon^e_{ij}, D_n) + \psi^p(\alpha_{ij}, r) \quad (2)$$

where α_{ij} and r are the flux related to the backstress and the cumulative plastic strain. The stress and the generalized thermodynamic force that conjugate to the damage vector are expressed as

$$\sigma_{ij} = \frac{\partial\psi^e}{\partial\varepsilon^e_{ij}} \Leftarrow \hat{\sigma}_{ij}(\varepsilon^e_{ij}, D_n) \quad (3)$$

and

$$Y_n = -\frac{\partial\psi^e}{\partial D_n} = G(a_n)\,\frac{\partial a_n}{\partial D_n} \equiv \bar{G}(D_n) \quad (4)$$

where the approximated relations in (1) are used to evaluate the stress for the current damage. The secant and tangent stiffness matrices in (3) can also be evaluated using numerical difference of the stress (1), once the damage vector is determined. It is clear from (4) that the thermodynamic force Y_n conjugate to the damage vector and is proportional to the energy release rate $G(a_n)$, and (a_n) is the physical (true) damage system, such as crack area.

The second group of generalized thermodynamic forces are associated with plastic deformations and are derived by

$$R = -\frac{\partial\psi^p}{\partial r} \text{ and } X_{ij} = -\frac{\partial\psi^p}{\partial\alpha_{ij}} \quad (5)$$

We assume the existence of a convex pseudodissipation potential that is separately decomposed of two functions: a yield function and a damage dissipation function (Lemaitre 1992; Voyiadjis and Kattan 1999; Maugin 2000). This is expressed as

$$F = F^p(\sigma_{ij}, \alpha_{ij}, R) + F^d(Y_n, D_n) \quad (6)$$

The rate of state variables associated with plastic deformation are generated in a similar manner to the classical plastic formulation

$$\dot{\varepsilon}^p_{ij} = \dot{\lambda}\,\frac{\partial F^p}{\partial\sigma_{ij}};\ \dot{\alpha}^p_{ij} = \dot{\lambda}\,\frac{\partial F^p}{\partial X_{ij}};\ \dot{r} = \dot{\lambda}\,\frac{\partial F^p}{\partial R} \quad (7)$$

In addition to (7), $\dot{\lambda}$ is evaluated using the consistency relation $dF^p = 0$.

The rate of damage is therefore written in a similar manner as

$$\dot{D}_n = \dot{\lambda}\,\frac{\partial F}{\partial Y_n} = \dot{\lambda}\,\frac{\partial F^d}{\partial Y_n} \tag{8}$$

The damage dissipation potential F^d is a convex function in terms of the thermodynamic damage-conjugate force, and $\dot{\lambda}_d$ is determined from the consistency equation $dF^d = 0$. A simple damage dissipation function for noninteracting crack systems can be expressed as

$$F^d = \frac{1}{2}\sum_{n=1}^{N} \eta_n^2 \langle \bar{G}_n(D_n) - \bar{G}_n^c \rangle^2$$

$$\langle \bar{G}_n - \bar{G}_n^c \rangle = \begin{cases} \bar{G}_n - \bar{G}_n^c, & \bar{G}_n - \bar{G}_n^c > 0 \\ 0, & \bar{G}_n - \bar{G}_n^c < 0 \end{cases} \tag{9}$$

where the ''damage material coefficients'' are η_n, viscosity-type coefficients that are very large for brittle fracture, and $\bar{G}^c$, the critical dissipated energy for damage system n.

Substituting (9) into (8) yields an uncoupled system of ordinary differential equations (ODE) that is needed in order to determine the current state of damage variables. More sophisticated damage dissipation functions can be assumed that will yield a coupled system of ODEs. In either case, the material damage coefficients need to be calibrated from material testing. This can be a very cumbersome task in case of multiple damage systems with a general dissipation function (interaction damage).

Once η_n and $\bar{G}^c$ are known, we need to determine the thermodynamic forces associated with damage $\bar{G}$ at every iteration as a function of D_n. This is done numerically in the proposed MMANN method

$$\bar{G}_n(D_n) = -\frac{\partial \psi^e}{\partial D_n} = -\frac{\partial}{\partial D_n}\int_0^{\varepsilon^e} \frac{\partial \psi^e}{\partial \varepsilon_{ij}^e}\,d\varepsilon_{ij}^e = \int_0^{\varepsilon^e} \frac{\partial \hat{\sigma}_{ij}(\varepsilon_{ij}^e, D_n)}{\partial D_n}\,d\varepsilon_{ij}^e \tag{10}$$

Next, a similar procedure must be carried out to select the form of the plastic yield function F^p, its variables and materials parameters. One suggestion is to generate another ANN to represent the yield function of the yield of the composite, which can include the average stress, a back stress, and effective plastic strain as input variables. Having said that, the definition of yield or ''average yield'' for a heterogeneous composite medium is not clear and needs to be defined. Another useful approach, that is well suited for the proposed MMANN method, is to assume that the plastic part of the dissipation may not need to have an exact definition of a yield similar to those defined for homogeneous materials. It can be, without loss of generality, a simple convex function that enables the derivation of the plastic strain from the average stress variables [first part of (7)].

Thus far, the formulation of the MMANN method can be readily applied to a medium with growing cracks, with the assumption of no plastic strain accumulations. This assumption is common and well suited for fiber reinforced plastic composites. The inelastic part application needs to be further developed by applying the above general formulation; this is beyond the scope of this paper. The damage modeling using the MMANN model can be implemented with a standard material subroutine in a displacement-based FE. In this case, the damage evolution is determined by solving the ODEs in (8).

CONCLUSIONS

A new method, termed micromechanical neural network models, is proposed to generate nonlinear micromechanical models for a periodic heterogeneous medium. The ANN is trained to approximate the equivalent response from detailed FE results of RVEs. Applications are presented and include nonlinear behavior as a result of matrix behavior and the singularity created by induced fiber/matrix interface cracks. The ability of MMANN models to generalize the overall behavior under different multiaxial strain paths and damage has been demonstrated. The highly inelastic response of MMC composites, especially in the presence of damage, has been chosen to demonstrate the ability of MMANN to predict overall nonlinear behavior of the material.

A relatively small number of training cases were used to achieve successful training. This computational premium is small compared to the number of inelastic constitutive calculations that are typically performed in a nonlinear structural analysis. Furthermore, it is doubtful that simple nonlinear micromechanical models can be formulated to achieve all shown aspects of the MMC response. The new method can be used for incorporating detailed material and damage models with microstructure into a structural level analysis. General derivation is also presented using TIP formulation and approximate responses from MMANN in order to define the response of an equivalent homogeneous medium with growing damage. This approach can be applied to model damage in FRP composites. Further development is needed in the case of inelastic nonmonotonic response of damaged heterogeneous materials, such as MMCs.

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